Traders’ Heterogeneity and Bubble-Crash Patterns in Experimental Asset Markets

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Abstract

We provide a heterogeneous agent model for experimental closed-book call-markets with speculators, fundamental and noise traders. We provide structural estimates of the parameters of the model using experimental data. The model allows us to identify the different types of traders empirically. We find that fundamental traders and speculators have higher cognitive abilities and terminal wealth than noise traders. More importantly, we find that all three types of traders are essential to explain the mechanics of bubbles and crashes. In the initial period, fundamental traders buy from noise traders. Next, speculators buy from fundamental traders during the boom. Finally, speculators generate the crash by selling to noise traders.

Keywords: Experimental Asset Markets, Bubbles, Trader Heterogeneity

JEL Classifications: C90, C91, D03, G02, G12

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1 Introduction

“There are exceptional people out there who are capable of starting epidemics. All you have to do is find them.”

There are several historical examples of bubbles: the Dutch tulip mania (1634-1637), the South Sea Company Bubble (1720), the Roaring Twenties stock-market bubble (1922-1929), the Dot-com bubble (1995-2000) and more recently, real-estate bubbles in the US as well as Europe and China. Bubbles generate price distortions that are potentially associated with allocative inefficiencies and have often led to financial crises. Thus, economists are naturally drawn towards studying bubbles via theoretical models and empirical methods. Laboratory experiments provide a useful tool to study bubbles empirically since they allow economists to control a variety of factors that are difficult to control in field environments (e.g., trading institutions, the fundamental value process and the dividend process).

Bubbles and crashes in experimental asset markets were first documented by Smith et al. [1988] (SSW) and proved to be a very robust result in experimental economics. Some authors blame bubbles on speculators (e.g., Smith et al. [1988], Ackert et al. [2006], Moinas and Pouget [2012], Haruvy and Noussair [2006]), while others suggest that subject confusion and heterogeneity are responsible for the observed price-swings (e.g., Lei et al. [2001], Kirchler et al. [2012], Caginalp and Ilieva [2005]). In general, a clear understanding of the mechanics of bubble formation is still missing. For instance, we do not have many models that help us to understand when and why bubbles start and crash (Brunnermeier 2008).

In order to fill this gap, we propose a heterogeneous agent model which sheds light on the mechanics of bubble formation in experimental closed-book call markets.

There are three classes of agents in the model: noise traders, fundamental traders and speculators. Noise traders are equally likely to be either buyers or sellers in each period, and their bid/ask price is determined by the previous period clearing price plus a noise term. Fundamental traders buy when the price is below and sell when the price is above the fundamental value. Speculators form their price expectations taking into account the presence of noise traders in the spirit of Level-1 traders. They then buy when the price is expected to increase and sell otherwise, i.e., their trading behavior is motivated by potential capital gains. We provide structural estimates of the parameters of the model.

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1 Gladwell [2002].
2 The bubble-crash pattern persists in treatments with capital gains taxes, no short selling constraints, transaction fees or the use of a sophisticated subject pool such as corporate managers, professional stock traders etc. (King et al. [1993], Lei et al. [2001]). Experience of traders in a stationary environment is one of the major factors which damps or eliminates bubbles under the SSW design (Smith et al. [1988], Porter and Smith, 1995, Dufwenberg et al. [2005], Hussam et al. [2008]).

Lei et al. [2001] show that even if capital gains are not possible, the standard bubble-crash pattern persists. Smith et al. [2000] show that if dividends are paid at the end of the trading horizon only (the least confusing design) the formation of bubbles is least likely. Similarly Kirchler et al. [2012] and Kirchler and Huber [2011] show that the main source for subject-confusion is the decreasing fundamental value process. Lei and Vesely [2009] show that a pre-trading period before the actual asset market experiment starts designed to decrease subject confusion about the stochastic dividend process entirely eliminates the bubble-crash pattern.

4 For a review of the links between agent-based models and human subject experiments, see Duffy 2006.

5 In contrast to Duffy’s and Unver’s near-zero-intelligence traders we do not need to assume that noise traders have weak foresight. This assumption is crucial for Duffy and Unver 2006 to generate the observed crash-patterns in the lab. In our model it is the interplay of the different trader types which generates the bubble-crash pattern.

6 We elaborate below that speculators are similar to Level-1 trader types, characterized by one step
using experimental data on five closed-book call market sessions. The estimation is conducted by fitting aggregate simulated variables—prices and volume—to the corresponding aggregate experimental variables.

We estimate that 12.5−33% of subjects are speculators, 22−33% of subjects behave like simulated fundamental traders and the remaining subjects behave like noise traders. We show that simulated fundamental traders accumulate assets early and sell their units gradually to speculators and noise traders. Speculators accumulate a substantial number of assets during the boom and initiate the crash. Simulated fundamental traders and speculators end up with much lower asset holdings (close to zero) than noise traders. Speculators end up with the the highest simulated terminal wealth levels, followed by fundamental traders. Noise traders end up with significantly lower wealth levels.

The model also allows us to identify traders’ types in the data. Remarkably, we obtain a very clear self-selection of subjects into types, which is consistent with individual characteristics of subjects. In particular, fundamental traders are much better in predicting the first-period price than other types, and noise traders are much worse in price forecasting during the crash compared to fundamental traders and speculators. Also, noise traders have much lower cognitive skills (measured by the Cognitive Reflection Test in Frederick [2005]; CRT) than other types.

Our model and estimation results are instrumental to understanding the mechanics of experimental bubbles and crashes. In the first period, noise traders under-predict the price compared to the fundamental value, since they form expectations based on the noise term only. As a result, they sell large amounts of the asset to the fundamental traders. After that, speculators buy aggressively (from fundamental and noise traders) since they predict an upward price trend and expect capital gains. At the peak, which is well above the fundamental value, speculators realize the upcoming crash and start selling massively to noise traders, who do not foresee the downward price trend. To summarize, noise traders are first taken advantage of by the fundamental traders and then during the crash by the speculators. Not surprisingly, they end up with the lowest terminal wealth compared to other classes of traders. Importantly, all three types of traders are essential for explaining the dynamics of the bubble.

Our work complements the existing literature. The main contribution of our paper, relative to the existing literature, is that we provide a framework that helps understand how bubbles are generated and crash in experimental asset markets. Specifically, we identify trading strategies that generate bubble-crash patterns. Relative to Caginalp and Ilieva [2005] and Caginalp and Merdan [2007], we emphasize modeling individual behavior to capture individual and aggregate features of the data. Duffy and Ünver [2006] are the first to propose an agent-based model with noise traders to generate bubble-crash patterns. The main departure from Duffy and Ünver [2006] is that we introduce heterogeneous agents and find significant differences in behavior between traders’ types. The mix of heterogeneous agents also allows us to dispense with the assumption of weak foresight of iterated reasoning in their expectation formation (Stahl and Wilson [1994, 1995], Costa-Gomes and Crawford [2006], Crawford et al. [2013]). In accordance with the Level-k literature we impose that their respective anchoring type (commonly referred to as L0-type) are noise traders. In contrast to Haruvy and Noussair [2006] we do not assume that speculators know the specific parameters, which characterize the behavior of the noise-traders. We assume that they only know the functional form of the equilibrium price process under a trader type distribution degenerate at the noise-trader types. Caginalp and Merdan [2007] also provide a heterogeneous agent model for asset markets, which could in principle be used to generate price dynamics, similar to the ones observed in the lab. However, their model is not suitable to identify whether individual traders indeed belong to certain trader classes ex-post. Hence we would not be able to assess, for instance, the cognitive ability levels of specific individuals, belonging to different groups.

Duffy and Ünver [2006] use the notion “near-zero-intelligence” traders.
(the probability of being a buyer decreases with time), which generates the crash in their environment. In our model, the crash is generated endogenously by the interplay of the three types of traders. Haruvy and Noussair [2006] also admit heterogeneous types and adapt the model of DeLong et al. [1990a] into the framework of experimental asset markets. Haruvy and Noussair [2006] focus on fitting price dynamics at the aggregate level, and are not interested in fitting trading volume paths. Our approach on the other hand allows us not only to fit trading prices, but also to fit trading volumes at both the individual and aggregate levels. These differences are essential to improve our understanding of the mechanics of bubbles and crashes. While a better fit of aggregate variables is a compelling feature of our model, it is not our prime objective. Fitting the trading volume at the individual level has substantive implications for the understanding of bubble formation. Indeed, our work sheds light on the questions of when and why bubbles start and crash in experimental asset markets.

Section 2 presents the experimental data. Section 3 presents our model and its main building blocks. In Section 4 we estimate the parameters of the model based on aggregate variables. We then proceed and use the estimates to identify different types of traders in the experimental data. Section 5 shows that bubbles and crashes are generated by the interplay of speculators, fundamental and noise traders. Section 6 investigates the out-of-sample predictive power of our model. Section 7 concludes the paper.

2 Experimental Design and Data

2.1 Experimental Design and Procedures

The experimental design builds on the seminal study of Smith et al. [1988]. In the laboratory market subjects had the opportunity to trade assets with a stochastic dividend process. The market had a finite time horizon of 15 periods. At the end of each period, each unit of the asset in a trader’s inventory paid an uncertain dividend of 0, 8, 28, or 60 francs (the experimental currency) with equal probability (e.g., Smith et al. [1988], Boening et al. [1993], Caginalp et al. [2000, 2001], Haruvy et al. [2007], Hussam et al. [2008]). Therefore, the expected value of the dividend payment in each period was 24 francs. It was publicly known that the dividend was independently drawn each period and the actual dividend paid in each period was the same for all traders.

Given the dividend process, the fundamental value of the asset could be calculated at any time within the experiment. More specifically, the fundamental value could be calculated as the expected value of the dividend in each period (24 francs) times the number of periods remaining (including the current period). The fundamental value of the asset was, therefore, declining from 360 francs in period 1 to 24 francs in period 15, and assets became worthless at the end of period 15. At the beginning of the experiment, each trader was endowed with two units of the asset and a cash balance of 2,000 francs. Traders had the opportunity to buy and sell assets in each period via a closed-book call market. Subjects could not purchase more units than they could afford nor sell more units than they had in their inventories, i.e., negative cash balances and short selling was not possible.

Another difference is that we also focus on a call-market trading institution.

Conditional on asset and cash constraints, subjects submitted buy and/or sell limit orders. Buy orders were ordered from highest to lowest and sell orders from lowest to highest. The price was determined by the intersection of these schedules. If they were overlapping, the lowest market clearing price possible was then determined to be the trading price in period t, at which transactions were executed. If no such price existed, i.e., if the entire demand schedule happened to be below the aggregate supply schedule the highest bid-price was reported to the subjects. No transactions were executed at this price.

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not allowed. Inventories of assets and cash balances were carried over from period to period. No interest was paid on cash holdings and there were no transaction costs.

At the beginning of each period traders also made forecasts of the transaction price for that period. They were paid for the accuracy of their forecasts. All earnings from forecasting accumulated in a separate account from the traders’ cash on hand, and thus these payments did not affect the market capital asset ratio.

At the beginning of each session, subjects were provided the instructions of the first task of the experiment. The instructions for all tasks were projected on an overhead. The first stage in all sessions consisted of a cognitive reflection test (Frederick [2005]) to measure the cognitive ability of all subjects. This stage was hand-run with the subjects providing their answers to the three questions on a decision sheet. Subjects were given as much time as they needed to complete the three questions. Subjects received two dollars for each correct answer at the end of the session. Once everyone finished, the decisions sheets were collected and the instructions for the second stage were handed out. The market instructions were read aloud in front of the subjects. Afterwards, the subjects were given five minutes to complete a short quiz. The experimenter went over the answers on an overhead and then started the market. The subjects were privately paid their earnings for all stages of the experiment. Throughout the experiment, subjects were encouraged to ask questions at any time. The questions were asked and addressed privately to avoid the possibility of biasing the entire group.

The experiment consisted of 5 markets conducted at Indiana University. In four out of the five sessions (Sessions 1, 3, 4 and 5) nine subjects participated in the experiment and eight subjects participated in Session 2. Subjects were recruited from undergraduate courses via the IELab Recruiting System. Many of the subjects had taken part in previous experiments in economics and other disciplines, but no subjects had participated in markets of comparable designs and each subject participated in only one market of this study. The markets were computerized and programmed with the z-Tree software package. At the end of a session, each subject’s final holdings of francs were converted to dollars at the predetermined and publicly known conversion rate of 148 francs to 1 US dollar. Each session lasted approximately 80 minutes including instructional period and payment of subjects. Subjects earned on average $24.

2.2 Data Description

In this section we focus on the main features of the experimental data. Figure 1 shows the experimental trading prices for the five sessions and the average price. The lower straight line in the graph depicts the fundamental value of the asset, whereas the higher straight line illustrates the maximum possible value of the asset. We observe that for every session the trading price exceeds this maximum value at least once. The price dynamics show the standard bubble-crash pattern (e.g., Smith et al. [1988], Boening et al. [1993], Caginalp et al. 2000, 2001, Haruvy et al. 2007, Hussam et al. 2008, and Williams [2008]). The standard bubble measures are presented in Table 2. In this paper we provide a model that generates price patterns, volume patterns, and bubble measures that are similar to the observed data.

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11 They were paid 50 francs for the forecast within 10%, 20 francs for within 25%, and 10 francs for within 50% of actual price. We followed Haruvy et al. [2007] for the forecast rewards structure.

12 The instructions for all stages of the experiment are available upon request.

13 See Fischbacher [2007] for a discussion of the z-Tree software package.
3 The Model

We propose a simulation model similar (in spirit) to models suggested by Duffy and ¨Unver [2006] and Haruvy and Noussair [2006]. We construct the model to resemble the laboratory economy described in Section 2.

In our market environment $N$ agents interact in $T$ periods and trade a single financial asset. Initially, each agent $i$ is endowed with $x_i^0$ units of cash and $y_i^0$ units of the financial asset. At the end of every period the asset pays random dividends drawn with equal probability from a commonly known support $\{d_1, d_2, d_3, d_4\}$, with $d_i \geq 0$ and $d_1 < d_2 < d_3 < d_4$. The expected dividend is denoted as $\bar{d} = \frac{1}{4} \sum_{i=1}^{4} d_i$. Since our model is supposed to fit the laboratory environment, the dividend support is $\{0, 8, 28, 60\}$. (In general, the support does not necessarily have to be restricted to four values or to an i.i.d. dividend process.) The fundamental value of the asset in every period is common knowledge and given by

$$FV_t = \bar{d}(T - t + 1) \text{ for } t = 1, \ldots, T.$$ 

Under rational expectations and risk neutrality, prices should equal the fundamental value.

3.1 Simulated Call Market Environment

In this section, we describe the market environment as well as agents’ behavior. In every trading period $t = 1, \ldots, T$ traders may either buy or sell units of the financial asset (or remain inactive). During the experiments, traders were allowed to submit both bids and asks simultaneously, potentially for multiple units. To capture this feature in the simulations, we subdivide each trading period into $S$ submission rounds.

In each of the $s = 1, \ldots, S$ submission rounds a trader is either a seller or a buyer (the decision or probability to be buyer or seller is described below). Trader $i$ in submission round $s$ and trading period $t$ can submit an ask price, $a_{s,t}^i$, for one unit of the asset if she is a seller and may submit a bid, $b_{s,t}^i$, if she is a buyer. These features of the model allow us to capture the fact that in the experiments, traders can submit both bids and asks in a given trading period and to track the volume of the traded asset during the

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14See also Gode and Sunder [1993] for goods markets.
experiments. Since we do not allow for short-selling or borrowing, in every trading period, traders cannot sell more units than in their holdings, and buyers cannot exceed their cash holdings.

In every submission round in trading period $t$ all the bids and ask prices are aggregated to obtain the price and quantity traded$^{[13]}$. That is, as in the experiments, the trading institution used to determine market clearing prices is a closed book call market$^{[16]}$. In each round, any buyer who submits a bid above the market-clearing price $p_{s,t}$ buys one unit of the asset at $p_{s,t}$. Thus, the updated cash holdings of a buyer are

$$x_{s,t}^i = x_{s-1,t}^i - p_{s,t}^i 1(b_{s,t}^i > p_{s,t}^i),$$

and similarly the updated unit holdings are

$$y_{s,t}^i = y_{s-1,t}^i + 1(b_{s,t}^i > p_{s,t}^i),$$

for each submission round $s$ and trading period $t$. The symbol $1_C$ is an indicator function, taking the value 1 if condition $C$ is satisfied and 0 otherwise. The assumptions on the behavior of traders will imply that prices across submission periods $p_{s_1,t}$ and $p_{s_2,t}$ with $s_1 \neq s_2$ will not vary systematically. This particular structure was chosen to account for the possibility of trading multiple units in a period and to fit the simulated trading volume to the data.

In each round, any seller who submits an ask below the market-clearing price $p_{s,t}$ sells one unit at $p_{s,t}$. The updated cash and unit holdings of a seller are

$$x_{s,t}^i = x_{s-1,t}^i + p_{s,t}^i 1(a_{s,t}^i < p_{s,t}^i)$$

and

$$y_{s,t}^i = y_{s-1,t}^i - 1(a_{s,t}^i < p_{s,t}^i)$$

for each submission round $s$ of trading period $t$.

Next, we explain how market-clearing prices are determined in each submission round. A market-clearing trading price is any price which satisfies

$$|B| = |\{i = 1, \ldots, N : b_{s,t}^i > p_{s,t}\}| = |A| = |\{i = 1, \ldots, N : a_{s,t}^i < p_{s,t}\}|.$$

That is, the market-clearing price is a price at which the number of buyers equals the number of sellers. Let $b_J$ be the lowest bid price in $B$ and $b_{J+1}$ be the highest bid price outside of $B$. Similarly, let $a_K$ be the highest ask in $A$ and $a_{K+1}$ be the lowest ask outside of $A$. Then, for any $\lambda \in (0, 1)$ the following price clears the market:

$$p_{s,t} = \lambda \min \{b_J, a_{K+1}\} + (1 - \lambda) \max \{b_{J+1}, a_K\}. \quad (1)$$

In the experiments, the market-clearing price was defined as the lowest price at which there was an equal number of sellers and buyers. We use $\lambda = 10^{-2}$ to determine the}

$^{[13]}$Bids are ordered from highest to lowest to obtain the inverse demand schedule, while asks are ordered from lowest to highest to obtain the inverse supply schedule. The trading price is determined as the intersection of the inverse demand and supply schedules. Note that we can have multiple market clearing prices within a period. The number of rounds fitting the data best is equal to 2, and the average simulated difference across submission prices within a period is very small, namely, 0.578.

$^{[16]}$In closed book call markets traders typically observe only the market clearing price (on their screens) but are not informed about the identities of the traders who sell or buy units of the asset. Typically they also do not observe the trading volume in this market environment.

$^{[17]}|U|$ denotes the cardinality of the set $U$. 7
corresponding simulation market-clearing price. Whenever the entire demand schedule happened to be below the supply schedule, the highest bid price was taken as a proxy for the market-clearing price. No trades are executed at this price during the simulations.

After the $S^{th}$ (last) submission round in trading period $t$, the random dividend $D_t$ is realized and traders update their cash holdings according to:

$$x^i_{1,t+1} = x^i_{S,t} + D_t y^i_{S,t}.$$ 

The period market-clearing price is defined as the average of the sequence of market-clearing prices $\{p^s_t\}_{s=1}^{20}$:

$$p_t = \frac{1}{S} \sum_s p^s_t.$$ 

The simulated market-clearing prices will then be compared with the observed market-clearing prices in Section 4.

### 3.2 Traders

We consider a model with heterogeneous types of traders, namely, noise traders, fundamental traders and speculators. We need at least three types of traders in order to understand the mechanics of bubbles and crashes (see Section 5). We start by describing the behavior of noise traders. We assume that there are $(N - k_1 - k_2)$ noise traders present in the market with $k_1, k_2 \in \{0, 1, ..., N\}$ and $k_1 + k_2 \leq N$. As in Duffy and Ünver [2006], we impose some assumptions on the behavior of noise traders and model their bidding behavior.

At the beginning of every submission round $s$ in every period $t$, each noise trader is a buyer with probability $\pi_t$ and a seller with probability $1 - \pi_t$. We assume that $\pi_t = \pi = 0.5$. We depart from Duffy and Ünver [2006] who assume that the probability of being a noise-buyer is decreasing over time and initially equal to 0.5. Specifically they assume that $\pi_t = \max\{0.5 - \phi t, 0\}$ with $\phi \in [0, 0.5]$. Duffy and Ünver [2006] refer to this as ‘weak foresight’ assumption. This assumption implies that as $t$ increases, the excess supply increases under weak foresight and generates a downturn of prices. Duffy and Ünver [2006] need this assumption to generate the crash pattern, given that their model consists of only noise traders. We do not need to impose the ‘weak foresight’ assumption since in our model the crash is endogenously generated by the interplay of different types of traders.

If a noise trader is selected to be a buyer in trading round $t$ and submission period $s$, she submits a bid subject to cash availability. Her bid is of the form

$$b^i_{s,t} = \min \{(1 - \alpha)\epsilon_t + \alpha p_{t-1}, x^i_{s,t}\},$$

where $\alpha \in [0, 1]$, $\epsilon_t \sim U[0, \kappa F V_t]$, $\kappa \geq 0$ is a parameter, $x^i_{s,t}$ denotes the current cash
holdings of agent \(i\), and \(p_{t-1}\) is the market-clearing price in period \(t-1\). No bid is submitted by a noise buyer whenever her cash holdings are zero, i.e., if \(x_{s,t}^i = 0\).

Similarly, if a noise trader is selected to be a seller in trading round \(t\) and submission period \(s\), she submits an ask subject to her unit holdings. Her ask is of the form

\[
a_{s,t}^{n,i} = (1 - \alpha)\epsilon_{t} + \alpha p_{t-1},
\]

where \(\alpha \in (0,1)\) and \(\epsilon_{t} \sim U[0, \kappa FV_{t}]\). No ask is submitted by the noise seller whenever her unit holdings are zero, i.e., if \(y_{s,t}^i = 0\).

That is, noise traders submit bids and asks based on the previous period price and a noise term. The parameter \(\alpha \in (0,1)\) captures the behavioral notion that anchoring effects may be important - in our context the relevant anchor is the market clearing unit price in the previous period.

If we ignore the possibility that unit or cash constraints can be binding, then it is straightforward to show that, if there are only noise traders, the market-clearing price converges to \(\frac{\kappa FV_{t}}{2}\). If \(\kappa = 2\), then market-clearing prices converge to the fundamental value. A value of \(\kappa > 2\) indicates that traders are willing to pay on average more than the fundamental value of the asset to obtain some units of it.

The fact that unit and cash constraints can be binding adds an additional layer to the problem, which may create substantial deviations from the fundamental value of the asset even if \(\kappa = 2\). To see this, notice that in a cash-rich environment (cash constraints never bind) with a relatively low number of units, prices tend to be higher than the fundamental value of the asset, creating a bubble through the endowment channel. The intuition is straightforward - suppose for simplicity that \(S = 1\): with low unit holdings the probability that some agent with zero asset holdings is selected as a seller in some trading round is positive. This could create aggregate excess demand, which can only be offset by a corresponding increase in prices. In other words, even an environment with only noise traders is sufficiently interesting to address some questions related to aggregate price variables, as also shown in [Duffy and Unver 2006] in the context of a double-auction trading institution.

However, adding \(k_2\) speculators and \(k_1\) fundamental traders provides new insights on price and volume dynamics and thus on bubble formation. Fundamental traders are a combination of classes of agents suggested by [Cason 1992] and [Haruvy and Noussair 2006].

[Cason 1992] provides a simulation model for goods markets where market-clearing prices are determined via a closed-book call market institution. In his model, buyers with random valuation \(v\) for the good submit bids which are randomly drawn from an interval \([l_t, v]\). Buyers adaptively update the lower bound of the interval according to \(l_t = (1 - \alpha^c)l_{t-1} + \alpha^c p_{t-1}\), where \(p_{t-1}\) is the market-clearing price in period \(t-1\) and the initial value \(l_0\) is the lowest feasible contract price.

Sellers with a random cost parameter \(c\) submit asks drawn from an interval \([c, a_t]\) with \(a_t = (1 - \beta)a_{t-1} + \beta p_{t-1}\), where \(a_0\) is the highest feasible contract price. For \(\beta, \alpha^c \in (0,1)\), Cason shows that the lower/upper bounds of the intervals converge to the competitive equilibrium price \(p_t\) if \(t \to \infty\). The rate of convergence and therefore the speed of learning is determined by \(\alpha^c\) and \(\beta\). The lower those two parameters the faster the convergence. Cason further shows that agents behaving according to this rule produce trading prices similar to the ones observed in experimental goods markets under a closed book call market structure resulting in high efficiency levels.

\[21\] We assume that \(p_0 = 0\). We will show below that this assumption does not affect our results significantly.
Haruvy and Noussair [2006] present a simulation model for double auctions asset markets. The authors define passive investors as traders who buy assets at the current standing ask price if that price is below the fundamental value. Similarly, they sell assets whenever the current standing bid price is above the fundamental value of the stock. The task for fundamental traders under a closed book call market structure is significantly more complex than under a double auction institution since at the point of the bid/ask submission, no bids and asks are observed.

We therefore had to develop a method to characterize the beliefs of fundamental traders about the period \( t \) market-clearing price given their limited information.

We modify the adaptive agents considered by Cason [1992] and the passive investors considered by Haruvy and Noussair [2006] to model adaptive fundamental or simply fundamental traders. A fundamental trader computes in every period the bound

\[
l_t = \alpha_f (l_{t-1} - \bar{d}) + (1 - \alpha_f) p_{t-1},
\]

with \( l_0 = FV_1 + \bar{d} \). The intuition is straightforward: \( l_t \) serves as a proxy for the expected market-clearing price in period \( t \), which is unknown by the agent at the time of the bid/ask submission. We subtract \( \bar{d} \) to control for the decreasing fundamental value and thus our updating rule for \( l_t \) differs from the one suggested by Cason [1992]. Notice that \( l_t \) does not depend on \( s \), i.e., we do not allow fundamental traders to learn within the period, which is in line with the experimental design. The functional form of \( l_t \) suggests that price expectations are formed adaptively. Empirical evidence for adaptive expectation formation, used by subjects during asset market experiments, is provided by Haruvy et al. [2007], Smith et al. [1988] and Williams [1987].

While noise traders are randomly determined to be buyers or sellers, a fundamental trader decides whether to be a buyer or a seller. If the bound is below the fundamental value, \( l_t \leq FV_t \), and her cash holdings are positive, \( x_{s,t} > 0 \), then she chooses to be a buyer and submits a bid as follows:

\[
B_{s,t} = \min \{ B_{s,t}, x_{s,t} \}
\]

\[
b_{f,i}^{s,t} = \min \{ B_{s,t}^{f,i}, x_{s,t} \}
\]

\[
b_{f,i}^{s,t} \sim U[l_t, FV_t].
\]

That is, if the trader believes that the market-clearing price is below the fundamental value, she submits a bid \( b_{f,i}^{s,t} \in [l_t, FV_t] \) if she has enough cash. If, on the other hand, the bound is above the fundamental value, \( l_t > FV_t \), and her asset holdings are positive, \( y_{s,t} > 0 \), then she chooses to be a seller and submits an ask from the interval between the fundamental value and the bound:

\[
B^{s,t} \sim U[l_t, FV_t].
\]

\[
a_{f,i}^{s,t} \sim U[FV_t, l_t].
\]

Bids and asks are random to allow for some decision errors. Naturally, if agents cash holdings are zero, \( x_{s,t} = 0 \), given \( l_t \leq FV_t \) she does not submit a bid, and if her asset holdings are zero, \( y_{s,t} = 0 \), given \( l_t > FV_t \), she does not submit an ask.

We also introduce \( k_2 \) speculators into the model. Speculator \( j \) decides whether to buy or sell assets based on her expectations about market clearing prices in period \( t \) and period \( t+1 \). Notice that a speculator has to form the relevant expectations before submitting an ask or bid order. If

\[
E_{t-1}^{i}(p_{t+1}) > E_{t-1}^{i}(p_t)
\]

speculator \( j \) decides to submit a bid order expecting to make capital gains by selling in the following period. At this point we do not further specify the expectations operator.

\[22\] Further, we show in section 11.1 that we cannot reject that there are no participants in our subject pool, behaving as if they were passive traders with rational expectations.
$E^j$ but emphasize that the expectations do not depend on the submission round $s$. A more detailed discussion follows below. Intentionally successful and profitable bids – from the perspective of the speculator – are in the interval $[E^j_{t-1}(p_t), E^j_{t-1}(p_{t+1})]$. Allowing for decision errors we specify that speculators submit bids of the following form:

$$b_{s,t}^{sp,i} = \min\{B_{s,t}, x_{s,t}\}$$

$$B_{s,t}^j \sim U[E^j_{t-1}(p_t), E^j_{t-1}(p_{t+1})]. \quad (6)$$

If on the other hand

$$E^j_{t-1}(p_{t+1}) \leq E^j_{t-1}(p_t)$$

speculator $j$ decides to sell and submits an ask order of the form:

$$a_{s,t}^{sp,i} \sim U[E^j_{t-1}(p_{t+1}), E^j_{t-1}(p_t)]. \quad (7)$$

We use a level-$k$ modeling approach to compute speculators’ expectations $^{23}$ Specifically we assume that speculators are level-1 traders, best responding against a benchmark population of level-0 noise traders. The average equilibrium price process in a population consisting solely of noise traders takes the form:

$$E_{t-1}(p_t) = \alpha p_{t-1} + (1-\alpha)\frac{\kappa}{2}FV_t$$

The assumption that speculators know the actual values of $\alpha$ and $\kappa$ is too strong, especially in a closed book call market. We therefore assume that speculator $j$ forms expectations in the following way:

$$E^j_{t-1}(p_t) = \gamma_1 p_{t-1} + \gamma_2 FV_t \quad (8)$$

with $\gamma_1 \in [0,1]$ and $\gamma_2 \geq 0$. Iterating (8) one period forward yields $E^j_{t-1}(p_{t+1})$ as a function of publicly observed variables, namely, $FV_t$, $FV_{t+1}$ and $p_{t-1}$. Note that speculators keep buying the asset as long as $^{24}

$$\gamma_1(1-\gamma_1)p_{t-1} < \gamma_2(\gamma_1(T-t+1)d - d). \quad (9)$$

Since the right hand side of (9) decreases monotonically over time, it is generally possible to find parameter-values under which speculators initially buy the asset and sell it towards the end of the trading horizon. Since fundamental traders sell their assets while prices increase well beyond the FV and noise traders follow the price trend, it is the change in the behavior of speculators, which induces the crash in our model.

We next summarize the simulations steps within a period $t$:

1. At the beginning of submission round $s \in \{1, ..., S\}$ each of the $(N-k_1-k_2)$ noise traders is determined to be a buyer or a seller according to the probability $\pi = 0.5$. Noise buyers submit bids and noise sellers submit asks according to the rules specified in (2) and (3).

$^{23}$We further impose that $a_{s,t}^{sp,i} = FV_t$ in the last two periods of the simulation, whenever a speculator is a seller.


$^{25}$Note that iterating the expected prices forward yields: $\gamma_1(\gamma_1 p_{t-1} + \gamma_2 FV_t) + \gamma_2 FV_{t+1}$ which is greater than $E_{t-1}(p_t)$ (in which case speculators buy) iff

$$\gamma_1(1-\gamma_1)p_{t-1} < \gamma_2(\gamma_1(T-t+1)d - d).$$

Using that $FV_{t+1} = FV_t - d$ and $FV_t = (T-t+1)d.$
2. Simultaneously, each of the \( k_1 \) fundamental traders computes \( l_t \) and decides to be either a buyer (if \( l_t \leq FV_t \)) or a seller (if \( l_t > FV_t \)). Depending on this decision, a fundamental trader submits either a bid or an ask according to the rules stated in (4) and (5).

3. Simultaneously, each of the \( k_2 \) speculators computes \( E^j_{t-1}(p_t) \) and \( E^j_{t-1}(p_{t+1}) \) based on (8) and decides to be either a buyer (if \( E^j_{t-1}(p_{t+1}) > E^j_{t-1}(p_t) \)) or a seller (if \( E^j_{t-1}(p_{t+1}) \leq E^j_{t-1}(p_t) \)). Depending on this decision, a speculator submits either a bid or an ask according to the rules stated in (6) and (7).

4. After all the bids and asks are submitted, the market-clearing price is computed. The market-clearing price is then reported (no additional information is revealed to the traders) and trades are executed. Traders’ cash and unit holdings are updated accordingly.

5. The process above is repeated for the \( S \) submission rounds. After trades in the \( S^{th} \) submission round are executed, the asset pays its random dividends and cash holdings are updated accordingly.

For a given set of parameters \( (S, \alpha, \kappa, \alpha^f, k_1, k_2, \gamma_1, \gamma_2) \) and experimental characteristics, \( N, T \) and endowments \( \left\{ x_i \right\}_{i=1}^N, \left\{ y_i \right\}_{i=1}^N \), each simulation run \( M \) of the model will generate a price sequence \( (p^M_1, p^M_2, ..., p^M_T) \), an \( N \times T \) matrix of cash holdings and an \( N \times T \) matrix of asset holdings. The results can be interpreted as the simulation-equivalent of one experimental session. We next show how to estimate the parameters of the model using our experimental data.

4 Model and Data: Estimation

In this section, we estimate the parameters of the structural model using only aggregate prices and quantities. Based on this optimal fit with respect to aggregate variables, we characterize the strategies used by speculators, fundamental and noise traders. The results are then used to identify the different types of traders in the data. We will show that there is a tight relationship between the terminal wealth levels of agents and their trading strategies in the simulations, which is also confirmed by the experimental data.

We start by obtaining estimates for the model parameters. Given the initial cash and asset-holdings endowment from the experimental design (2000 francs and 2 units), the number of traders \( N \) and the number of trading periods \( T \), the model presented in Section 2 consists of eight free parameters: the number of submission rounds \( S \), the anchoring parameter of the noise traders \( \alpha \), the noise-support parameter \( \kappa \) of the noise traders, the learning speed parameter of the fundamental traders \( \alpha^f \), the expectations parameters for speculators \( \gamma_1 \) and \( \gamma_2 \), the number of fundamental traders \( k_1 \) and the number of speculators \( k_2 \). We imposed the restriction that

\[
\alpha^f = \alpha
\]

to reduce the dimensionality of the parameters. In order to obtain parameter estimates, we specified a grid for the parameters \( S, k_1 \) and \( k_2 \) and used an interior-point algorithm ([Byrd et al. 1999, 2000, Waltz et al. 2006]) to estimate the remaining parameters by

---

26Earlier estimation results indicated that for unrestricted estimations \( \alpha \) and \( \alpha^f \) tend to be very similar in magnitude.

27\( S \in \{1, 2, 3, 4, 5\}, k_1, k_2 \in \{1, 2, 3, ..., 9\} \). We used several hundreds of initial starting values to ensure that we achieved a global minimum.
minimizing the following objective function:

\[
SSE(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2) = \\
\sum_{t=1}^{15} \sum_{i=1}^{5} \left( \frac{\bar{p}(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)^M - p_{i,t}^E}{FV_1} \right)^2 + \sum_{t=1}^{15} \sum_{i=1}^{5} \left( \frac{\bar{Q}(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)^M_t - Q_{i,t}^E}{TSU} \right)^2,
\]

where \( \bar{p}(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)^M \) is the average simulated asset price in period \( t \) using \( M = 100 \) and some values for the parameters \((S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)\), \( p_{i,t}^E \) is the price of the asset in period \( t \) and session \( i \). Similarly, \( \bar{Q}(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)^M_t \) denotes the average simulated trading volume in period \( t \), and \( Q_{i,t}^E \) is the observed trading volume in period \( t \) and session \( i \). Equation (10) computes the sum of the squared differences of the simulated average variables to the corresponding observed variables. Notice that all our parameters are identifiable since we have ten observables (the price and quantity data from five sessions) to estimate seven parameters.

Table 1: Estimation Results

<table>
<thead>
<tr>
<th>( \tilde{\alpha} )</th>
<th>( \tilde{\kappa} )</th>
<th>( \tilde{S} )</th>
<th>( \tilde{\gamma}_1 )</th>
<th>( \tilde{\gamma}_2 )</th>
<th>( \tilde{k}_1 )</th>
<th>( \tilde{k}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.820</td>
<td>4.915</td>
<td>2</td>
<td>0.256</td>
<td>0.036</td>
<td>22%</td>
<td>22%</td>
</tr>
</tbody>
</table>

Table 1 provides the “best fit” parameter combination. According to Table 1, 22% of the traders in our model behave like fundamental traders who learn the market-clearing price process relatively slowly (\( \tilde{\alpha} = 0.820 \)). Note that the estimated parameter-value is very close to the corresponding value for \( \alpha \) provided by Duffy and Unver 2006 (0.848), who consider a model with noise traders only. The noise amplitude parameter \( \kappa \), which measures the degree of confusion among noise-traders, takes a value of 4.915 and is again very close to the corresponding parameter estimate provided by Duffy and Unver 2006 (4.085). However, we also observe that there are not only noise-traders in our data. 44% of our traders are either speculators or fundamental traders. Based on the parameter estimates for \( \gamma_1 \) and \( \gamma_2 \), we assert that speculators anchor their expectations towards the previous prices but less than noise traders and do not put a large weight on the fundamental value of the asset. Importantly, we fit the aggregate variables of the model to the data and do not restrict any behavior of the agents nor do we directly fit the individual behavior of simulated agents to the observed behavior of experimental subjects.

\(^{28}\)We normalized the former squared difference by the fundamental value of period 1 and the latter difference by the total stock of units (TSU). Sessions 1,3,4 and 5 had a TSU of 18 and session 2 had a TSU of 16 since the number of agents in session 2 is eight instead of nine. We used TSU = 18 to obtain parameter estimates, which produce prices and quantities during the simulations, which fit the data. That is, we treat session 2 as if it was generated by nine traders. A different approach would be to normalize by the average TSU across sessions – the difference in the results is negligible.

\(^{29}\)See page 13 in Duffy and Unver 2006.

\(^{30}\)Again, see page 13 in Duffy and Unver 2006.

\(^{31}\)Our results are robust to relaxing the assumption of \( p_0 = 0 \). If we ignore the first period price and volume data our adjusted objective function becomes

\[
SSE(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2) = \\
5 \sum_{i=1}^{5} \sum_{t=2}^{15} \left( \frac{\bar{p}(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)^M_t - p_{i,t}^E}{FV_2} \right)^2 + 5 \sum_{i=1}^{5} \sum_{t=2}^{15} \left( \frac{\bar{Q}(S, \alpha, \gamma_1, \gamma_2, \kappa, k_1, k_2)^M_t - Q_{i,t}^E}{TSU} \right)^2
\]

with \( FV_2 = 336 \). We re-estimated the resulting objective function and do not find substantial differences in the resulting parameter estimates.
Table 2 provides several bubble measures computed using the data and the model. A quick look at the simulated bubble measures reveals that they are indeed close to the actual data measures. Figure 2 also depicts the simulated and actual prices and quantities.

Surprisingly, there are only two models that provide a theoretical framework for bubbles in experimental asset markets. In what follows we discuss how we contribute to this literature. Duffy and Ünver [2006] and Haruvy and Noussair [2006] propose agent-based models which provide a good fit to the aggregate price data. In addition to fitting crucial elements of the data,22 we would like to emphasize that our model is built to understand which trading strategies generate bubble-crash patterns in experimental asset markets.

Overall, our model explains various features of the data better than a model with only near-zero intelligence (NZI) traders modeled as in Duffy and Ünver [2006]. Duffy and Ünver [2006] allowed for weak foresight assuming that \( \pi_t = \max\{0.5 - \phi t, 0\} \). The relevant parameters of a model with only noise traders in a closed book call market environment are \((S, \phi, \kappa, \alpha)\). We performed the same estimation procedure as above to obtain estimates for the relevant parameters and to obtain a closed book call market analogue of the model consider by Duffy and Ünver [2006]. In order to test which model provides a better fit, we conducted two different tests which focus on average prices. First, we tested for cointegration of the simulated average prices and the observed average prices using a Johansen [1991] approach.33 We cannot reject the null hypothesis that the average simulated prices from our model are cointegrated with the observed average prices (p-value < 0.05). On the other hand, the prices stemming from the model-specification with only NZI traders are not cointegrated with the actual price data at a 10% level. This reveals that prices in the model with only NZI traders and observed prices follow independent random walks. We acknowledge that this result might be driven by the small sample size. Hence we assumed a common stochastic trend between the two time series and performed a standard J-test (Davidson and MacKinnon [1981]) to compare our model with a model, which considers only NZI traders as in Duffy and Ünver [2006]. This approach suggests

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22Prices, trading volume and final wealth distributions.

33We cannot reject the null hypothesis that either of the simulated average prices nor the observed average prices follow a unit root process using a standard augmented Dickey Fuller test.
Table 2: Bubble Measures: Call Markets, 5 Sessions

<table>
<thead>
<tr>
<th>Bubble Measure</th>
<th>Formula</th>
<th>Data Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turnover</td>
<td>$\sum_{t=1}^{15} g_t/TSU$</td>
<td>Mean (Std. Err) with Fund. &amp; Speculators</td>
</tr>
<tr>
<td>Amplitude</td>
<td>$\max_t \left{ \frac{(P_t-FV_t)}{P_t} \right} - \min_t \left{ \frac{(P_t-FV_t)}{P_t} \right}$</td>
<td>5.11 (3.9)</td>
</tr>
<tr>
<td>Total Dispersion</td>
<td>$\sum_{t=1}^{15}</td>
<td>\text{median}(P_t) - FV_t</td>
</tr>
<tr>
<td>Average Bias</td>
<td>$\frac{1}{T}\sum_{t=1}^{15} (P_t - FV_t)$</td>
<td>180.55 (65.96)</td>
</tr>
<tr>
<td>APD</td>
<td>$\frac{1}{T}\sum_{t=1}^{15}</td>
<td>P_t - FV_t</td>
</tr>
<tr>
<td>PD</td>
<td>$\frac{1}{T}\sum_{t=1}^{15}</td>
<td>P_t - FV_t</td>
</tr>
<tr>
<td>RAD</td>
<td>$\frac{1}{T}\sum_{t=1}^{15}</td>
<td>P_t - FV_t</td>
</tr>
<tr>
<td>RD</td>
<td>$\frac{1}{T}\sum_{t=1}^{15}</td>
<td>P_t - FV_t</td>
</tr>
<tr>
<td>RPAD</td>
<td>$\frac{1}{T}\sum_{t=1}^{15}</td>
<td>P_t - FV_t</td>
</tr>
<tr>
<td>Haessel</td>
<td>$R^2$ of OLS regression: $P_t = \alpha + \beta FV_t + \epsilon_t$</td>
<td>0.29 (0.25)</td>
</tr>
<tr>
<td>Boom</td>
<td>$\max \left{ N : (P_{j+k})<em>{k=0}^{N} &gt;&gt; (FV</em>{j+k})_{k=0}^{N} \right}$</td>
<td>13.2 (1.3)</td>
</tr>
<tr>
<td>Trend Dur.</td>
<td>$\max \left{ N : P_t &lt; P_{t+1} - FV_{t+1} &lt; ... &lt; P_{t-(N-1)} - FV_{t+N-1} \right}$</td>
<td>5.8 (2.86)</td>
</tr>
</tbody>
</table>

that we can reject a model with NZI traders only in favor of our model at a 1% (p-value = 0.004)\(^{34}\) Out of sample predictions of our model and their fits—using the estimates from Table I—are presented in Section 6.

Our model also improves upon the heterogeneous trader-types model used by Haruvy and Noussair [2006], which is based on the standard setup introduced by DeLong et al. [1990a]. First, Haruvy and Noussair [2006] significantly underestimate the observed trading volume in 20 out of their 22 sessions (compare the Turnover between Tables II and III against Table VIII in their paper). We show below that an accurate estimation of the turnover and dynamic patterns of the trading volume is essential for our understanding of bubbles and crashes. Second, their model does not generate final wealth distributions across trader types which mimic the actual wealth distributions, indicating that some behavioral aspects are missing in their model.

Lastly, the model considered by Haruvy and Noussair [2006] cannot be used to analyze the impact of different trading institutions on trading strategies. Within every simulation-period, Haruvy and Noussair [2006] use a Tâtonnement algorithm to find the unique market clearing price through quantity alterations and fit this price to the average observed trading price stemming from double auction experiments. Lugovskyy et al. [2011] show that a change of trading institutions from double-auctions to Tâtonnement affects behavior and may reduce bubbles. In other words, bubbles and crashes are not institution-invariant and it is desirable to construct a model, which may address this issue. While the model suggested by Haruvy and Noussair [2006] does not account for those differences, our model can be extended easily to analyze the impact of trading institutions in future work.

4.1 Simulated and Experimental Strategy Comparison

In this section, we use our estimates (see Table I) to compare the simulated individual behavior to the actual behavior of traders. The main message of this section is that the model does a good job at describing individual data, even though the parameters are

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\(^{34}\) See Appendix C for details.
estimated using aggregate data. In particular, the model also predicts patterns for asset holdings and terminal wealth levels for each type of trader which are consistent with the data.

Figure 3 illustrates the simulated asset holdings of the three classes of traders. We observe that fundamental traders initially accumulate assets and then gradually sell their units while prices are above the fundamental value of the stock. Speculators accumulate assets while they predict upward trending trading prices. When speculators predict a decrease in prices they start selling their assets to noise traders. It is important to see that speculators do not only predict the turning point of prices but also generate the crash in prices due to their massive sales decision. Their decisions have a substantial impact since they keep accumulating assets during the upward trend, and thus can influence trading prices. Figure 3b depicts the individual asset holding paths in an environment consisting only of NZI traders in the call market analogue of [Duffy and Unver 2006]. Clearly, the two models provide different predictions for traders’ asset holdings.

We use the simulated trading strategies to identify trader types on a micro-level using individual asset holding data. For each trader $i$ in session $s$ we let $u_{i,t}^s$ be trader $i$’s stock of assets in period $t$. We further denote with $\bar{u}_t^S$ the average period $t$ asset holdings of simulated speculators. Analogously we define by $\bar{u}_t^F$ and $\bar{u}_t^N$ the average asset holdings of simulated fundamental and noise traders. For classification purposes we ran for every trader two OLS regressions described in (11):

$$u_{i,t}^s = \beta_0^k + \beta_1^k \bar{u}_t^k + \epsilon_{i,t}^s \quad k \in \{S, F\}.$$  

(11)

In the next step, we tested for every trader whether $\beta_1^F > 0$ or $\beta_1^S > 0$ at a 5% level. We never encountered a case in which both $\beta_1^F$ and $\beta_1^S$ were strictly greater than zero. Whenever $\beta_1^F$ was strictly greater than zero at a 5% level for subject $i$, we classified that trader as a fundamental trader. A similar logic applied for the identification of speculators. Whenever for subject $i$ neither $\beta_1^F$ nor $\beta_1^S$ were strictly greater than zero, we classified this subject as a noise trader. Table 3 shows the number of identified speculators, fundamental-

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35Controlling for heteroscedasticity.
and noise traders for every session. We observe that the fractions of fundamental traders and speculators are relatively stable across sessions. Overall we identify eleven fundamental traders (25%) and ten speculators (22%) in the data, confirming the accuracy of our previous estimation, which was based on aggregate variables.

Figure 4a shows the simulated asset holding dynamics and Figure 4b depicts the asset holding dynamics of the three representative speculators, fundamental and noise traders from the data.\textsuperscript{36} Note that the slopes of the representative traders lines mimic the slopes of the simulated traders asset-holdings lines quite well. We will show below that it is the behavior of speculators which causes the crash of asset prices. It is therefore not surprising that in session number two –in which we don’t observe a crash– the number of speculators in the data is the lowest.

Next, we show that our model provides good predictions for terminal wealth levels of all three types of traders. The simulated speculators, fundamental and noise traders make on average 3919.6, 3149.8 and 2036 francs respectively. Table 3 reports the median terminal wealth levels of the three trader types. The median terminal wealth levels of speculators and fundamental traders in the data are not significantly different from the simulated terminal wealth levels of the speculators and fundamental traders, respectively (Mann-Whitney test p-value (Fundamental): 0.25; (Speculators): p-value $\approx 1$).\textsuperscript{37} The median terminal wealth of the remaining subjects is also not significantly different from the simulated terminal wealth levels of the noise traders (Mann-Whitney test p-value $\approx 1$). Furthermore, the wealth levels of the subjects identified as fundamental traders and speculators are significantly higher than the wealth levels of the remaining subjects at a 1% level using a Mann-Whitney Test (p-value < 0.001). The terminal wealth levels of speculators in the data are significantly higher than the terminal wealth levels of fundamental traders at a 1% level (Mann-Whitney test p-value $\approx 0$).

\textsuperscript{36}We selected the identified speculators, fundamental traders and their asset holdings over the course of the experiment. We generated “representative speculators” and “representative fundamental traders” by averaging their (trader-group specific) actual asset holdings for each period $t$. Similarly we constructed a “representative noise trader” by averaging over the asset holdings of the non-speculators and non-fundamental traders for every period $t$.

\textsuperscript{37}We performed a one-sample Mann-Whitney test for every empirical trader group and tested whether the respective median wealth levels are equal to the point predictions from the model.
In summary both speculators and fundamental traders outperform the remaining traders in terms of terminal wealth, and these results are significant at a 1% level. Moreover, speculators end up with higher terminal wealth levels than fundamental traders. The difference is also significant at a 1%.

Table 3 also presents the median number of correct answers provided by each trader group in the Frederick [2005] CRT test. The results are shown in column “CRT”. The median CRT scores are not statistically different between speculators and fundamental traders (Mann-Whitney p-value: 0.91). Also, fundamental traders and speculators outperform the noise traders in terms of CRT scores at 5% and 1% level, respectively (Mann-Whitney p-value (Fundamental): 0.013; Mann-Whitney p-value (Speculators): 0.008). That is, speculators and fundamental traders have higher cognitive abilities than noise traders and make higher earnings during the experiments.

Speculators and fundamental traders also have more accurate price forecasts than identified noise traders. We computed for every subject i who participated in session s, the accuracy of her period t price-forecast as the absolute deviation of her price forecast and the realized trading price. We find that the average period t price-forecasts of fundamental traders and speculators across sessions are significantly more precise than the price forecasts of noise traders at a 5% level (Mann-Whitney p-values: 0.045 (Fundamental) and 0.035 (Speculators)). Across the 15 trading periods speculators’ price-forecasts are not significantly better than the price forecasts of fundamental traders (p-value: 0.31).

Table 3: Trader Types Summary Statistics

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Speculators</th>
<th>Fundamental</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22%</td>
<td>22%</td>
<td>56%</td>
</tr>
<tr>
<td>2</td>
<td>12.5%</td>
<td>12.5%</td>
<td>75%</td>
</tr>
<tr>
<td>3</td>
<td>22%</td>
<td>33%</td>
<td>45%</td>
</tr>
<tr>
<td>4</td>
<td>22%</td>
<td>33%</td>
<td>45%</td>
</tr>
<tr>
<td>5</td>
<td>33%</td>
<td>22%</td>
<td>45%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>CRT</th>
<th>Wealth</th>
<th>CRT</th>
<th>Wealth</th>
<th>CRT</th>
<th>Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>2</td>
<td>3104</td>
<td>2</td>
<td>2895</td>
<td>1</td>
<td>2397</td>
</tr>
<tr>
<td>Average</td>
<td>1.8</td>
<td>3138</td>
<td>1.82</td>
<td>2997</td>
<td>0.7</td>
<td>2272</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>(1.1)</td>
<td>(711)</td>
<td>(1.3)</td>
<td>(349)</td>
<td>(0.7)</td>
<td>(839)</td>
</tr>
</tbody>
</table>

5 When and Why Bubbles Start and Crash?

5.1 The Mechanics of Bubbles and Crashes

The decomposition of traders performed above allows us to discuss when and why bubbles start and crash. Specifically, we are interested in the following questions. Who is responsible for the emergence and amplification of bubbles? Who generates the crash?

We computed the average asset holdings for each session and trader group. That is, we computed the session-specific asset-holding paths of representative fundamental traders, \( \bar{u}_{i,t}^F \), speculators, \( \bar{u}_{i,t}^S \), and noise traders, \( \bar{u}_{i,t}^N \). Approximately 50% of our subjects belonged to the last group and the remaining 50% of our subjects belonged either to the first or second group. We denote with i the sessions and with t the trading periods.

Second, we subdivided the trading horizon over all our sessions into several periods of interest: the first trading period, the boom periods, the price-peak period, and the

\[ E_{t-1}(P_t^s) - P_t^s \] \text{ for } t=1. We compared for every group of traders the average of these accuracy measures over the entire trading horizon.

\[ \text{Session-specific peak periods are 3, 12, 8, 10, and 7.} \]
Finally we analyzed the trading patterns of the three groups of traders within these periods of interest. We summarize our findings in Figure 6 and in a series of observations.

Observation 5.1 (Initial Accumulation). In period 1 only fundamental traders do not under-predict the price, which turns out to be slightly below the fundamental value. As a result, in this period fundamental traders substantially increase their asset holdings.

Figure 7 indicates that in the first period fundamental traders submit higher and more accurate price forecasts than speculators and noise traders (this difference is significant at a 1% level of a Mann-Whitney test). As a result, fundamental traders accumulate shares of the undervalued asset by buying mainly from noise traders. The average asset-holdings increase of fundamental traders amounts to 85% of the initial endowment. Noise traders sell on average 31% of their shares.

Observation 5.2 (Boom). In the periods between the Initial Accumulation and the session specific price peaks, speculators increase their asset holdings on average by 135%. Noise traders increase their asset holdings on average by 33%. Fundamental traders decrease their asset holdings on average by 60%.

Observation 5.2 indicates that speculators (and to a minor extent noise traders) fuel the bubble by buying shares from fundamental traders. During this process speculators gain substantial market power, since they hold on average around 25% of all the shares available.

Observation 5.3 (Peak). At price peaks, fundamental traders are unsuccessful in selling their shares and speculators decrease their asset holdings on average by 17%. Noise traders increase their asset holdings on average by 18%.

We measure the accuracy of price predictions by the deviation of the individual price forecasts from the actual trading prices. For the Mann-Whitney test, we took the median individual price forecasts deviation for each group of traders and each session. As a result we have 5 observations for each type of traders for each period. Figure 7 plots the medians of these 5 observations for each group and period.
In three out of five sessions, we observe that the representative fundamental traders do not change their asset holdings at the price peak. Given that fundamental traders actually want to sell their shares, we may assert that they are not successful in selling. We also observe that speculators decrease their average asset holdings only by 17%. They sell their assets essentially only to noise traders.

**Observation 5.4 (Crash).** Only noise traders systematically over-predict prices during the crash. Thus, during the crash only noise traders buy shares mainly from speculators.

As illustrated by Figure 6 during the crash periods, noise traders significantly over-predict the prices compared to fundamental traders and speculators (Mann-Whitney test shows that both results are significant at a 10% level).

Observation 5.4 is based on the fact that we observe a steady decline in the asset holdings of speculators and fundamental traders between the peak and the last trading period. In the periods after the peak and period $T - 1$, fundamental traders reduce their average asset holdings from 0.9 to 0.3 units on average and speculators reduce their asset holdings from 3.3 to 0.3 units. Only noise traders buy shares during the crash period. The median asset holdings of speculators and fundamental traders in period 15 are 0.3, whereas the median asset holdings of noise traders in period 15 is 3.65. This evidence, in conjunction with the fact that fundamental traders are rather inactive in the latter periods (including the peak), indicates that the group of speculators substantially benefits from their comparative advantage in forecasting trading prices during the crash.

This section shows that bubbles and crashes are generated by the interplay between speculators, noise and fundamental traders. First, fundamental traders buy from noise traders. Next, speculators buy from fundamental traders during the boom and sell their shares to noise traders during the crash. Notice though that speculation is only profitable in this environment due to the presence of noise traders who are willing to buy the overvalued asset in later periods.

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*In particular, periods 12 to 15.*
Figure 7: In and out-of-sample model predictions vs. data. Dashed Lines: Simulations (Lower Dashed Line: New Endowments (10, 10,000), Higher Dashed Line: Old Endowments (2, 2,000)). Solid Lines: Price-Data (Lower Solid Line: New Endowments (10, 10,000), Higher Solid Line: Old Endowments (2, 2,000)). Straight Solid Line: FV.

6 Comparative Statics and Out of Sample Predictions

In this section we test the predictive power of our model using a different data set. We use our estimated model to predict behavior in another experiment and then compare it with the actual data. In our initial experimental design we endowed subjects with 2 shares and 2000 francs. We ran five additional closed book call market sessions with different subjects, in which we only changed the initial endowments to 10 shares and 10,000 francs, keeping the cash/asset ratio constant. Note that we do not re-estimate the model parameters to best fit the price and trading volume paths. Instead, we simulated the market environment with the new endowments using the previously estimated parameters (see Table 1). This provides a more rigorous test of our model.

Figure 7 shows the simulated and actual (average) trading prices under the original endowments (2,2000) as well as the simulated and actual (average) trading prices under the modified endowments (10,10,000) Table 4 provides a comparison of simulated and actual bubble-measures under the new endowments.

We observe that our model provides accurate comparative statics: a proportional increase in asset and cash-endowments, which keeps the asset/cash ratio constant reduces the size of bubbles in experimental closed book call market significantly. Intuitively, the probability that noise traders run out of shares and thus cannot sell when they happen to be sellers is lower when they are endowed with more shares. Note that in both environments, traders never run out of cash. That is, overall, aggregate supply is effectively higher in the treatment with high endowments. Our model underestimates the new trading volume (not surprising since we kept $S = 2$) but it predicts correctly that the turnover decreases under higher endowments. We also observe that the model over-predicts actual prices, suggesting that additional behavioral changes, associated with the change in endowments, affect the resulting price dynamics.

We used the method described above to identify the different types of traders in the

\[^{42}\text{In every session nine inexperienced subjects participated.}\]
data. That is, we used the average simulated trading strategies stemming from the original experimental design (2,2000) to ensure a proper out-of-sample analysis. The session-specific type distributions are given in Table 5. Overall, out of the 45 subjects, we identify 11 speculators (24%) and 16 fundamental traders (38%) – a distribution quite similar to the one derived under the original design. The simulated terminal wealth levels of fundamental traders, speculators and noise traders are 15.434, 13.765 and 12.991 francs respectively. The simulated terminal wealth distribution indicates that fundamental traders earn significantly more money than speculators and noise traders in this environment. Speculators only make slightly more money than noise traders. This is not that surprising since bubbles are smaller under high endowments, thus implying reduced opportunities for capital gains. Empirically we observe a wealth distribution which is similar to the simulated one: fundamental traders end up with significantly higher terminal wealth levels than speculators at a 1% level (Mann-Whitney test, p-value: 0.005). Fundamental traders make also more money than noise traders. The latter difference is significant at a 5% level. The terminal wealth levels of noise traders and speculators are not significantly different at a 5% level (Mann-Whitney p-value: 0.07). Similar to our finding above we observe that the median wealth levels of speculators, fundamental and noise traders are not significantly different to the simulated final wealths (p-values: Speculators: 0.67; Fundamental: 0.45; Noise: 0.73).

We observe again that fundamental traders perform better than noise traders on the CRT at a 10% level (Mann-Whitney p-value: 0.067). The difference between the CRT-performances of fundamental traders and speculators is not significant (p-value: 0.47). Furthermore, speculators and fundamental traders perform jointly better on the CRT than noise traders at a 10% level (p-value: 0.09).

Table 5: Trader Types Summary Statistics: Out of sample

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Speculators</th>
<th>Fundamental</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22%</td>
<td>44%</td>
<td>34%</td>
</tr>
<tr>
<td>2</td>
<td>33%</td>
<td>33%</td>
<td>34%</td>
</tr>
<tr>
<td>3</td>
<td>22%</td>
<td>44%</td>
<td>34%</td>
</tr>
<tr>
<td>4</td>
<td>11%</td>
<td>33%</td>
<td>56%</td>
</tr>
<tr>
<td>5</td>
<td>33%</td>
<td>22%</td>
<td>45%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CRT Wealth</th>
<th>CRT Wealth</th>
<th>CRT Wealth</th>
<th>CRT Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>1.5</td>
<td>12030</td>
<td>2</td>
</tr>
<tr>
<td>Average</td>
<td>1.6</td>
<td>11440</td>
<td>2</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>(1.35)</td>
<td>(1856)</td>
<td>(1.08)</td>
</tr>
</tbody>
</table>

Overall, the out-of-sample analysis suggests that our model can be used to investigate how changes in the environment affect the dynamics of bubble formation. It can also be used to guide the design of policies that may help to dampen or eliminate bubbles.
7 Conclusion

This paper provides a heterogeneous agent model for experimental closed-book call-markets with heterogeneous agents, namely, with speculators, fundamental and noise traders. We estimate the parameters of the model using experimental data. The model allows us to identify the different types of traders in the data. We find that fundamental traders and speculators have both higher cognitive abilities and terminal wealth than noise traders.

Furthermore, we find that all three types of traders are essential to explain the mechanics of bubbles and crashes. Specifically, fundamental traders buy from noise traders in initial periods initiating an upward trend in prices. Next, speculators buy from fundamental traders during the boom. Finally, speculation is only profitable in this environment due to the presence of noise traders who are willing to buy the overvalued asset in later periods.

Our model can be used to obtain out-of-sample predictions. For instance, it can be used to analyze how changes in the environment (e.g., changes in the dividend and fundamental value processes, trading institutions, experience level, etc.) affect the dynamics of bubble formation. It can also be used to guide the design of policies that may help dampen bubble formation.

References


### 8 Appendix A

The Frederick cognitive reflection test consists of the following three questions (answers in brackets):

- A bat and a ball cost $1.10 in total. The bat costs $1 more than the ball. How much does the ball cost? (5)
- If it takes five machines five minutes to make five widgets, how long would it take 100 machines to make 100 widgets? (5)
- In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half the lake? (47)

Frederick ([Frederick](2005)) shows that the number of correct answers on the previous three questions are positively correlated with subject-specific results on other cognitive ability tests such as the Wonderlic Personnel Test (WPT)\(^{43}\) and the "need for cognition

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\(^{43}\)A 12-minute, 50-item test used by the National Football League and other employers to assess the intellectual abilities of their prospective hires.
scale” (NFC)\textsuperscript{44}. Frederick also shows that there is a tight correlation between CRT scores and scores of subjects on the Scholastic Achievement Test (SAT) and the American College Test (ACT). Table\textsuperscript{6} shows the CRT score distributions over the 5 sessions. The Table reports the fractions of subjects answering one, two, three or none of the questions correctly, the average number of correct answers as well as the median number of correct answers. We observe that typically about 2/3 of the subjects answer 0-1 questions correctly and around 1/3 of the subjects answer 2-3 questions correctly. The last row in Table\textsuperscript{6} shows the CRT score distribution reported in [Frederick 2005]. Frederick’s sample consisted of 3428 students. We observe a very similar distribution indicating that our sample is fairly representative.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Session/N Correct Answers & Zero & One & Two & Three & Average & Median  \\
\hline
\textbf{All} & 0.318 & 0.318 & 0.136 & 0.227 & 1.27 & 1  \\
S1 & 0.56 & 0.22 & 0.11 & 0.11 & 0.78 & 0  \\
S2 & 0.125 & 0.5 & 0.125 & 0.25 & 1.5 & 1  \\
S3 & 0.44 & 0.22 & 0.11 & 0.22 & 1.11 & 1  \\
S4 & 0.11 & 0.45 & 0.11 & 0.33 & 1.67 & 1  \\
S5 & 0.34 & 0.22 & 0.22 & 0.22 & 1.33 & 1  \\
\textbf{Frederick 2005} & 0.33 & 0.28 & 0.23 & 0.17 & 1.24 & 1  \\
\hline
\end{tabular}
\caption{Performances Cognitive Ability Test}
\end{table}

9 Appendix B

9.1 Exogenous Wealth Predictors

We also used exogenous measures to characterize determinants of final wealth levels. We use the Frederick Cognitive Reflection Test (CRT\textsuperscript{45}) results and the accuracy of initial price predictions as empirical terminal-wealth predictors.

Table\textsuperscript{7} columns one and two, show two standard rank correlation measures between the number of correct answers given by the agents in the CRT across sessions and their final wealth levels. The suggested correlation ranges between 0.24 and 0.31, depending on the correlation-measure used and they are significant on a 5% level\textsuperscript{47}. Although the magnitude of the correlation is low, the Frederick CRT results have some power to predict terminal wealth levels. The higher the number of correct answers given by subjects in the test, the higher their terminal wealth levels tend to be.

Table\textsuperscript{7} (columns three and four) also shows the two standard correlation coefficients,\textsuperscript{46}

\textsuperscript{44}An 18-item test, which measures the endorsement of statements like “the notion of thinking abstractly is appealing to me (Cacioppo, Petty and Kao (1984))

\textsuperscript{45}It is important to point out that the definition of final wealth does not include earnings from forecast accuracy and Frederick cognitive reflection test.

\textsuperscript{46}This test was developed by Frederick (2005). See Appendix A for more details.

\textsuperscript{47}Since the specific dividend paths could affect terminal wealth levels we followed also a secondary approach: For each subject we divided their individual terminal wealth levels, $x_T$, by the session-specific aggregate terminal wealth levels, $\sum_i x_T^i$. The correlation between the resulting fractions, $\frac{x_T}{\sum_i x_T^i}$, and the corresponding CRT results remain fairly unchanged and significant on a 5 - 6% level. This procedure controls for session specific dividend realization paths: A high dividend realization at an early trading period may affect the price dynamics differently than a high dividend realization at a later point in time.
measuring the relationship between the terminal wealth level of subjects across experiments and a prediction-accuracy measure. The prediction accuracy measure for every subject is computed as the absolute value of the difference between the price prediction of every individual in period 1 and the fundamental value of the asset in period 1. Similarly one could use the realized price in period 1 as a benchmark to evaluate the accuracy of the price prediction in period 1. The analogous results using the period one prices as benchmark are shown in columns five and six. The results indicate that both the Kendall and the Spearman correlation measures are significant and negative on a 5% level. The results are intuitive: better initial price predictions, reflecting a better understanding of the instructions and the market environment, result in higher terminal wealth levels.

Table 7: Terminal Wealth Predictors*

| Correct Answers | $|Prediction_{1} - FV_{1}|$ | $|Prediction_{1} - P_{1}|$ |
|-----------------|--------------------------|--------------------------|
|                 | Kendall | Spearman | Kendall | Spearman | Kendall | Spearman |
|                 | 0.24    | 0.31     | -0.22   | -0.33    | -0.21   | -0.30    |
|                 | (0.04)  | (0.04)   | (0.04)  | (0.03)   | (0.05)  | (0.05)   |

*P-values in parentheses

The two previous results indicate that both the CRT results and initial price forecast accuracy measures are suitable empirical predictors for terminal wealth levels. This suggests that the initial price forecast accuracy and the number of correct answers should also show a significant and negative correlation. Table 8 provides support for this conjecture. Indeed the correlation is significant on a 1% level indicating that the accuracy of the initial price prediction and the Frederick CRT elicit cognitive abilities of subjects on a similar quality level.

Table 8: Correlation correct answers and prediction accuracy in period 1*

<table>
<thead>
<tr>
<th></th>
<th>Kendall</th>
<th>Spearman</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr. (# Correct Answers, $</td>
<td>Prediction_{1} - P_{1}</td>
<td>$)</td>
</tr>
<tr>
<td></td>
<td>(&lt; 0.01)</td>
<td>(&lt; 0.01)</td>
</tr>
<tr>
<td>Corr. (# Correct Answers, $</td>
<td>Prediction_{1} - FV_{1}</td>
<td>$)</td>
</tr>
<tr>
<td></td>
<td>(&lt; 0.01)</td>
<td>(&lt; 0.01)</td>
</tr>
</tbody>
</table>

*P-values in parentheses

10 Appendix C

To perform the J-test we ran the following regressions:

(SFN) :  \( \bar{p}_t^E = \beta_0 + \beta_1 \tilde{p}_t + \epsilon_t^1 \)

(NZI) :  \( \bar{p}_t^E = \beta_0 + \beta_1 \tilde{p}_t + \epsilon_t^2 \),

where \( \bar{p}_t^E \) is the average observed price in period \( t \), \( \tilde{p}_t \) is the period \( t \)'s price stemming from the simulations which includes speculators, fundamental and noise traders (SFN), and \( \tilde{p}_t \) is the period \( t \)'s price stemming from the simulations which includes only NZI traders as

\[\text{Similar to the procedure used for the previous Table we re-calculated the correlations using the session-wide normalized terminal wealth levels as an experimental performance measure. This procedure preserves the significance on a 5% level.\]
The fit of SFN specification, measured via $R^2$, exceeds the fit of the NZI specification (94% versus 79%). However, a direct comparison via the $R^2$'s is not possible since the models are not nested. We therefore conducted a standard J-test for non-nested models, following Davidson and MacKinnon [1981], to decide between the two specifications. Controlling for heteroscedasticity we can reject the NZI specification in favor of the SFN specification (our model) at a 1% level (p-value = 0.007).

11 Appendix D: Individual Behavior: DeLong et al. [1990a]

In what follows, we analyze our data through the lenses of the well-established model of DeLong et al. [1990a] (DSSW) as applied to experimental markets by Haruvy and Noussair [2006] (HN). The main point is to show that our model provides a better framework to understand the data than the DSSW model. DSSW present a heterogeneous agent model consisting of trend-chasers or feedback traders, fundamental or passive traders and rational speculators.

Following DSSW and Haruvy and Noussair [2006] (HN), the functional forms of the demand functions, $D(.)$, of trend chasers (feedback traders), fundamental or passive traders (FV traders) and speculators are given in equations (12), (13) and (14) respectively:

$$D(p_{t-1}, p_{t-2}) = -\delta + \beta(p_{t-1} - p_{t-2}) \quad (12)$$

$$D(p_t) = -\alpha(p_t - FV_t) \quad (13)$$

$$D(p_{t+1}, p_t) = \gamma(E(p_{t+1}) - p_t) \quad (14)$$

where $\delta$, $\beta$, $\alpha$ and $\gamma$ are all non-negative parameters, $E(.)$ is the expectations-operator, $FV_t$ is the fundamental value of the asset in period $t$, and $p_t$ is the uniform transaction price in period $t$.

We use exactly the same method as HN to classify every subject (for every session separately) into one of the three types. For instance, if the change in a subjects’ asset holdings does not have the opposite sign as the difference of the transaction prices in period $t - 1$ and $t - 2$, that subject is classified as a trend-chaser for that particular period. Similarly, if the difference between the fundamental value of the asset in period $t$ and the transaction price in period $t$ does not have the opposite sign as the change in a subjects’ asset holdings in period $t$, he or she is classified as fundamental trader in period $t$. Lastly, a subject is classified as speculator in period $t$ if the difference between the average transaction price in period $t + 1$ and the average transaction price in period $t$ does not have the opposite sign as the change in a subjects’ asset holdings for that period.

Finally, the number of periods in which each subject belongs to every single type are counted, yielding a vector of three scores for every participant. A subject is classified as agent of the type for which he has the highest score, provided that the score is greater than or equal to $\tau = 8$. If a subject has a maximum score lower than 8, that subject does not belong to any of the three types and is classified as “other”. If a subject’s maximum score is the same for two (three) types and is higher than 8, the participant is assigned a weight of $1/2$ ($1/3$) to each type for which he has the same maximum score.

When applying this approach to our data 0.337 0.394 0.269 0 we obtain the overall type distributions very similar to the trader type distributions in Table VI of Haruvy.

49Their model provides a theoretical framework to investigate the potential consequences of destabilizing rational speculation in financial markets. For similar and additional follow up contributions see also Hart and Kreps [1986], DeLong et al. [1990b], Cutler et al. [1990], Hirshleifer [2001], Abreu and Brunnermeier [2003], Brunnermeier and Pedersen [2009].

50Note that speculators are assumed to have perfect foresight.
and Noussair [2006]. Namely we identify 33.7% of subjects as Passive traders, 39.4% as Feedback traders, 26.9% as Speculators, and 0% as Others.

Next, we discuss the robustness of the trader type classifications mentioned above when a notion of statistical significance is introduced.

As a motivating example for doing so, consider a session in which prices increase in every period and remain above the fundamental value in the last period. Further, consider a subject who neither buys nor sells assets in $T - 1$ periods but sells some shares in period $T$. Based on the method of HN, the subject would be equally classified as speculator, passive trader and feedback trader in $T - 1$ of the $T$ periods. Since the subject in our example sells in period $T$—prices were rising in every period and exceed the fundamental value— he would be classified as a passive trader in the last period. In other words, although that participant’s classification is ambiguous for $T - 1$ out of $T$ periods, the scoring rule presented by HN would still uniquely assign the subject into the fundamental trader type class.

To address this concern and to examine the robustness of the classification rule presented by HN, we test the correlations between changes in individual asset holdings and the relevant price/fundamental value differences.

For instance, if there is significant positive correlation between changes in trader $i$’s asset holdings and the difference between fundamental values and prices in every period, we cannot reject the null hypothesis that trader $i$ is a fundamental trader. Similarly, if there is significant positive correlation between changes in trader $i$’s asset holdings and the difference between prices in period $t - 1$ and period $t - 2$, we cannot reject the null hypothesis that trader $i$ is a feedback trader. Lastly, if there is significant positive correlation between changes in trader $i$’s asset holdings and the differences between prices in period $t + 1$ and period $t$, we cannot reject the null hypothesis that trader $i$ is a speculator. Whenever we cannot reject that a subject belongs to more than one type, the participant is assigned equal weight to each type he may belong to. Like before, whenever a participant does not belong to any of three types—either because there is no positive or no significant correlation—we classify him as “other”. This approach is a natural extension of the method described by HN and introduces a notion of significance into their framework.

For every session and participant we compute non-parametric Spearman-correlations between changes in individual asset holdings and the relevant price/fundamental value differences at varying significance levels. The resulting trader type distributions are presented in the lower panel of Table 9 (Method III).

Imposing a significance level of 5%, we observe that the method of HN significantly classifies around 16% of all traders into one of the three types, whereas 84% are classified as “other”. Even after relaxing the significance level to 25%, only 32% of all traders can be classified into one of the three trader-types, whereas 68% remain “others”. Lastly, we observe that the smallest fraction of traders are fundamental traders, independently of the significance level imposed.

The robustness concerns and fairly large unexplainable behavioral components are not the only reasons why we present an alternative method to identify different trader types in the following sections.

Another reason is the difference in institutional frameworks. While HN classify their participants into DSSW-types under a double auction trading environment, we consider a closed book call market.

To clarify why trading institutions may matter in this context, consider the classification of fundamental or passive traders. In the model of DSSW, passive traders buy assets

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51 We also compute Kendall-correlations but the results discussed below change only marginally.
Table 9: Trader Type Classification based on DeLong et al. 1990a and Haruvy and Noussair 2006

<table>
<thead>
<tr>
<th></th>
<th>Passive</th>
<th>Feedback</th>
<th>Speculators</th>
<th>Others</th>
<th>Passive</th>
<th>Feedback</th>
<th>Speculators</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Method: Haruvy &amp; Noussair I</strong> (Dropping first two periods and last)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>0.364</td>
<td>0.295</td>
<td>0.341</td>
<td>0</td>
<td>0.364</td>
<td>0.284</td>
<td>0.330</td>
<td>0.023</td>
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<td>CRT Average</td>
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<td>1.556</td>
<td>2</td>
<td>1.200</td>
<td>1</td>
<td>1.556</td>
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</tr>
<tr>
<td>CRT Median</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td><strong>Method: Haruvy &amp; Noussair II</strong> (Including first two periods and last)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>0.330</td>
<td>0.284</td>
<td>0.314</td>
<td>0.068</td>
<td>0.330</td>
<td>0.284</td>
<td>0.330</td>
<td>0.220</td>
</tr>
<tr>
<td>CRT Average</td>
<td>1.222</td>
<td>1</td>
<td>1.556</td>
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<td>1.400</td>
<td>1.667</td>
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<tr>
<td>CRT Median</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.500</td>
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<tr>
<td><strong>Method: Haruvy &amp; Noussair III</strong> (Significance test; Including first two periods and last)</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Distribution</td>
<td>0.337</td>
<td>0.394</td>
<td>0.269</td>
<td>0</td>
<td>0.337</td>
<td>0.394</td>
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<td>0</td>
</tr>
<tr>
<td>CRT Average</td>
<td>1.222</td>
<td>0.833</td>
<td>1.286</td>
<td>1</td>
<td>1.222</td>
<td>0.833</td>
<td>1.286</td>
<td>0</td>
</tr>
<tr>
<td>CRT Median</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

whenever the fundamental value in period $t$ exceeds the price in period $t$. However, under a closed book call market the period $t$ transaction price is ex-ante unobservable.

As a matter of fact, in a closed book call market, passive traders would have to form expectations about transaction prices in period $t$, before they submit their limit orders. The HN-classification-method approximates individual expectations by actual trading prices, assuming that passive traders have perfect foresight. The same argument holds for speculators, whose expectations about transaction prices in period $t+1$ are also approximated by actual transaction prices. Put differently, while rational expectations may be a good approximation for actual expectations under double auctions, it might be a worse approximation for closed book call markets.\(^{52}\)

11.1 **DeLong et al. 1990a +ε**

In this section we provide additional evidence, showing that the DSSW model and its adapted version presented by Haruvy and Noussair 2006 does not generate price processes, which fit our data.

To address goodness of fit questions, we first estimate the type distribution and the associated demand parameters following the method of HN. HN estimate the type distribution and the demand parameters by minimizing the root mean square error (RMSE) between average session- and simulated prices stemming from their model.

Simulating prices based on the demand functions in (12), (13) and (14) requires some assumptions on the expectation formation process of speculators. HN solve this potential issue in their RMSE-minimization algorithm via a level-$k$ model structure,\(^{53}\) assuming that speculators form expectations in two steps:

\(^{52}\)There is plenty of evidence that the rational expectations hypothesis does not hold in laboratory asset markets with uniform market clearing prices.\(^{53}\)See Nagel 1995, Nagel and Duffy 1997, Camerer et al. 2001, Nagel and Grosskopf 2008, Crawford and Iriberri 2007, Crawford et al. 2013.
In the first step, speculators believe that the price in period \( t + 1 \) equals the fundamental value. Based on this prior, speculators compute the resulting equilibrium prices for each period, taking their own price effect into account. In the second step, speculators use the resulting price-sequence as their price expectations for every period.

Before we use the method of HN and present our estimation results, we device a simple statistical test to check whether our data is indeed generated by a model, which is consistent with the assumptions of DSSW and HN.

Following the approach of Bossaerts et al. [2007], who present a CAPM+\( \epsilon \) model, we first rewrite the demand functions in (12), (13) and (14) as:

\[
\tilde{D}_{i,t}(.) = D_{i,t}(.) + \epsilon_{i,t},
\]

where \( i \) represents one of the three types from above. We add a mean-zero-noise-term, \( \epsilon_{i,t} \), to the demand functions of every type, capturing random factors which may affect individual demands. Along the lines of Bossaerts et al. [2007], we assume that these noise terms are identically and independently distributed (iid) over time but not necessarily across types.

Next, letting \( N_1, N_2 \) and \( N_3 \) be the fractions of feedback traders, passive traders and speculators respectively (which sum to one), we compute the equilibrium prices as:

\[
p_t = \frac{N_1 \delta}{N_2 \alpha + N_3 \gamma} + \frac{N_2 \alpha}{N_2 \alpha + N_3 \gamma} FV_t + \frac{N_3 \gamma}{N_2 \alpha + N_3 \gamma} \mathbb{E}p_{t+1} - \frac{N_1 \beta}{N_2 \alpha + N_3 \gamma} p_{t-1} + \frac{N_1 \beta}{N_2 \alpha + N_3 \gamma} FV_{t+1} - \frac{N_1 \beta}{N_2 \alpha + N_3 \gamma} p_{t-1} + f(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}),
\]

where \( f(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}) \) is a mean zero, iid random noise term. Using the belief formation structure of HN, the first step equilibrium prices are

\[
p_t = \frac{N_1 \delta}{N_2 \alpha + N_3 \gamma} + \frac{N_2 \alpha}{N_2 \alpha + N_3 \gamma} FV_t + \frac{N_3 \gamma}{N_2 \alpha + N_3 \gamma} FV_{t+1} - \frac{N_1 \beta}{N_2 \alpha + N_3 \gamma} p_{t-1} + \frac{N_1 \beta}{N_2 \alpha + N_3 \gamma} p_{t-2} + f(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})
\]

Using the observation that in experimental asset markets the fundamental value decreases in every period by the average dividend-payment, \( \bar{\delta} \), iterating \( \delta \) one period forward and taking expectations, we obtain the following level-1 expectations for speculators:

\[
\mathbb{E}p_{t+1} = FV_t + \frac{N_1 \delta - N_3 \gamma \bar{\delta}}{N_2 \alpha + N_3 \gamma} - \bar{d} + \frac{N_1 \beta}{N_2 \alpha + N_3 \gamma} (p_{t-1} - p_t)
\]

Plugging (17) into (15) and collecting terms yields a price process of the form:

\[
p_t = m_0 + m_1 FV_t + m_2 p_{t-1} + m_3 p_{t-2} + \eta_t,
\]

where

\[
m_0 = \frac{1}{1 + \frac{N_3 \gamma N_1 \beta}{(N_2 \alpha + N_3 \gamma)^2}}, \quad m_1 = \frac{1}{1 + \frac{N_3 \gamma N_1 \beta}{(N_2 \alpha + N_3 \gamma)^2}}.
\]

\[
\text{54 i.e.: } FV_{t+k} = FV_t - \bar{d}k.
\]
\[ m_2 = \frac{1}{1 + \frac{N_3^\gamma N_1^\beta}{(N_2^\alpha + N_3^\gamma)^2}} - N_1^\beta \left( \frac{N_3^\gamma}{N_2^\alpha + N_3^\gamma} - 1 \right) \]

\[ m_3 = \frac{N_1^\beta}{1 + \frac{N_3^\gamma N_1^\beta}{(N_2^\alpha + N_3^\gamma)^2}} \]

and \( \eta_t \) is an iid noise term with mean zero. Based on the non-negativity constraints on the parameters in the models of DSSW and HN, the coefficient \( m_2 \) has to be non-positive, whereas the coefficient \( m_3 \) has to be non-negative. In summary the validity of the set of assumptions, imposed by DSSW and HN can be tested via the following null hypotheses:

\[ H_0: m_2 \leq 0, \ m_3 \geq 0 \quad (19) \]

To test (19) we take first differences of the variables in (18) and estimate the coefficients \( m_2 \) and \( m_3 \) via standard dynamic panel methods.\(^{55}\)

Columns (4) and (5) of Table 10 show the Arellano-Bond\(^{56}\) (AB) and Blundell-Bond\(^{57}\) (BB) estimates for \( m_2 \) and \( m_3 \) respectively. Note that for the AB estimates we can reject that \( m_2 \) is non-positive whereas for the BB-estimates we can reject that \( m_3 \) is non-negative (5% significance level).\(^{58}\)

Since we can reject the joint hypothesis in (19), we conclude that the DSSW model does not generate price sequences, which are consistent with our data.

To investigate why the DSSW model does not fit our data, we estimate the structural parameters of the model, following the method presented in HN.\(^{59}\) The parameter estimates and the type distributions are shown in the first row of Table 11. Note that the estimates indicate that \( \alpha \) equals zero, rendering the actual fraction of fundamental traders, which are supposed to have rational expectations about trading prices in period \( t \), unidentifiable. Moreover, since the fractions of all other trader types depend on the estimated fractions of fundamental traders, their relative proportions are not identifiable either.

In a second estimation step we therefore dropped passive traders, whose behavior – as specified in the models of DSSW and HN – does not seem to contribute to our data generating process. The resulting estimates are shown in the second row of Table 11. However, this partial identification strategy is problematic, since the model of DSSW requires all three trader types for the emergence of bubbles.

Put differently, we know that we are missing at least one relevant type so that our observed bubbles emerge but we don’t know its relative proportion or how the type actually behaves. Nevertheless, we can assert that out of the identifiable traders, around 72% are feedback traders and about 28% are speculators (second row of Table 11).

Our results – based on aggregate data – suggest that there are no participants in our subject pool who can be identified as fundamental traders with rational expectations. Note that this finding is consistent with our results from Method III in Table 9. There, we show that among all three trader classes the fraction of fundamental traders is always the smallest. To further investigate that it is the lack of FV traders, which is responsible for rejecting the DSSW model for our data, we re-estimate the parameters along the lines of HN, fixing the trader type distribution.\(^{60}\)

\(^{55}\)Note that the first difference of FV is constant and therefore dropped from the estimation.

\(^{56}\)See Arellano and Bond [1991].

\(^{57}\)See Blundell and Bond [1998].

\(^{58}\)Columns (1)-(3) show the fixed effect, random effects and between estimates for \( m_2 \) and \( m_3 \) respectively.

\(^{59}\)We minimize the RMSE using Byrd et al. [1999, 2000], Waltz et al. [2006].
We use the trader type distribution under Method I, with $\tau = 8$ from Table 9 – which corresponds to the method used by HN– but mention that other trader type distributions from the same Table yield similar result. The estimation results for the demand parameters, restricting the type distributions, are shown in the third row of Table 11. Note that the optimal $\alpha$ is again estimated to be zero, which is simply another way of setting the effect of FV traders to zero in the model.

Table 10: Testing DeLong et al [1990a] $\epsilon$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Random Effects</td>
<td>Fixed Effects</td>
<td>Between</td>
<td>Arellano, Bond</td>
<td>Blundell, Bond</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.401**</td>
<td>0.402**</td>
<td>0.0742</td>
<td>0.502**</td>
<td>-0.221*</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.000)</td>
<td>(0.864)</td>
<td>(0.010)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>$m_3$</td>
<td>-0.00425</td>
<td>0.000358</td>
<td>0.534</td>
<td>-0.0603</td>
<td>-0.259**</td>
</tr>
<tr>
<td></td>
<td>(0.945)</td>
<td>(0.995)</td>
<td>(0.296)</td>
<td>(0.529)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$N$</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>59</td>
<td>63</td>
</tr>
</tbody>
</table>

*p-values in parentheses

* $p < 0.10$, ** $p < 0.05$

Table 11: Estimation Results; Method: Haruvy and Noussair [2006] (Minimizing RMSE).

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{\alpha}$</th>
<th>$\tilde{\gamma}$</th>
<th>$\tilde{\delta}$</th>
<th>$\tilde{\beta}$</th>
<th>$\tilde{N}_1$</th>
<th>$\tilde{N}_2$</th>
<th>$\tilde{N}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4182</td>
<td>5.9224</td>
<td>0.0228</td>
<td>0.4878</td>
<td>0.4608</td>
<td>0.0514</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>0.1476</td>
<td>7.4101</td>
<td>0.0268</td>
<td>0.7282</td>
<td>-</td>
<td>0.2718</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.0643</td>
<td>7.4871</td>
<td>0.0288</td>
<td>0.364</td>
<td>0.295</td>
<td>0.341</td>
<td></td>
</tr>
</tbody>
</table>

In summary, we conclude that the model of DSSW and its adapted version presented by HN does not generate prices, which are consistent with our data. We identify the lack of FV-traders with rational expectations about period $t$ trading prices as one of the reasons, why we can reject the model of DSSW and HN in favor of potential alternatives.